PDFs with a quantum computer

from arXiv:2011.13934

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18th January 2021, QTI TH meeting, CERN.

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Introduction

Parton density functions

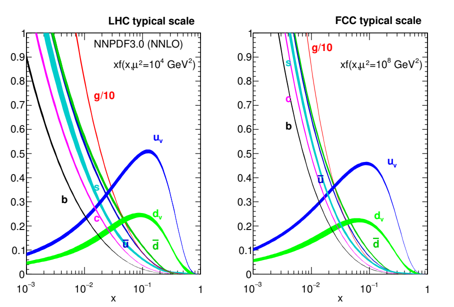
PDFs are essential for a realistic computation of hadronic observables:

$$\underbrace{\sigma_X(s, M_X^2)}_{Y} = \sum_{a,b} \int_{x_{\min}}^{1} dx_1 dx_2 \; \underbrace{\hat{\sigma}_{a,b}(x_1, x_2, s, M_X^2)}_{X} \; f_a(x_1, M_X^2) f_b(x_2, M_X^2)$$

PDFs are extracted by comparing theoretical predictions to real data.

$$X \otimes f \to Y$$

PDF determination is a regression problem.



Why Quantum Machine Learning?

Can we parametrize PDFs using a quantum computer?

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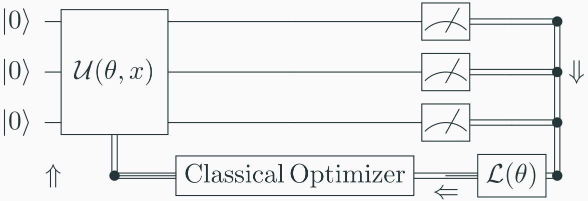
How?

Using variational quantum circuits and data re-uploading algorithms:



Problem is to identify a "circuit architecture" which:

- predicts PDF values,
- is trained with classical optimizer (VQC).

Why Quantum Machine Learning?

Why qPDF?

1. Proof-of-concept, study new architectures.
2. Obtain a hardware representation (analogy with GPU and FPGA).
3. Lower power consumption fits.

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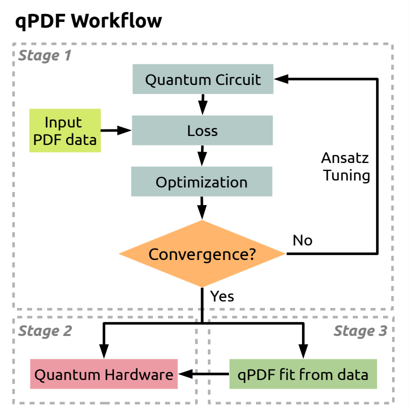
NISQ era Warning...

- Quantum devices implement few qubits, noise is a bottleneck.
- We can simulate quantum computation on classical hardware.

Building a qPDF model

Steps of the work

1. Define circuit Ansätze
2. Test it against reference PDF data.
3. If convergence is achieved, perform a global PDF fit.
4. Test model in a quantum device.



Summary of techniques

Circuit

$$\mathcal{U}(\theta, x) |0\rangle^{\otimes n} = |\psi(\theta, x)\rangle$$

Hamiltonian

$$Z_i = \bigotimes_{j=0}^{n} Z^{\delta_{ij}}$$

Measurement

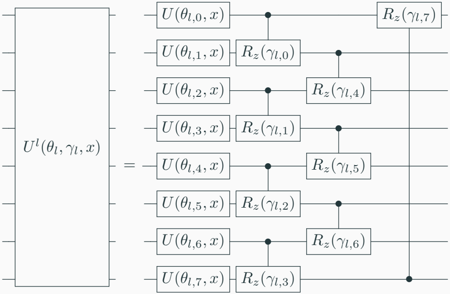
$$z_i(\theta, x) = \langle\psi(\theta, x)| Z_i |\psi(\theta, x)\rangle$$

Relation to PDF

$$\mathsf{qPDF}_i(x, Q_0, \theta) = \frac{1 - z_i(\theta, x)}{1 + z_i(\theta, x)}$$

Ansätze

We consider one global Ansatz with different building blocks



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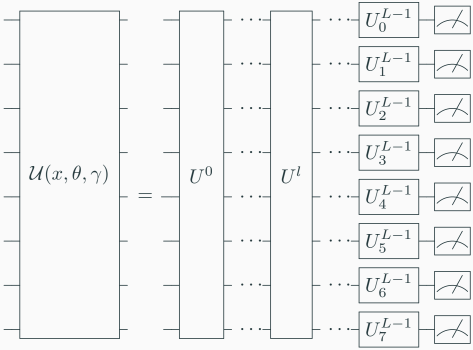


Ansätze

Weighted Ansatz

$$U_w(\alpha, x) = R_z(\alpha_3 \log(x) + \alpha_4) R_y(\alpha_1 x + \alpha_2)$$

Fourier Ansatz

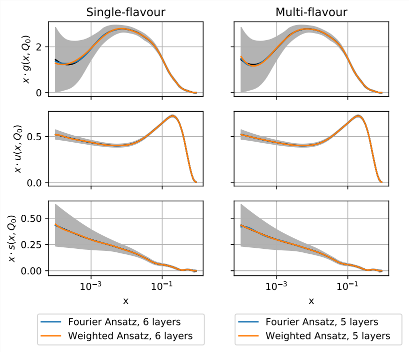
$$U_f^{(}\alpha, x) = R_y(\alpha_4) R_z(\alpha_3) R_y(-\pi/2 \log x) R_y(\alpha_2) R_z(\alpha_1) R_y(\pi x)$$

Linear and logarithmic scale

Both Ansätze include $(x, \log x)$ dependency.

Stage 1: Ansatz tuning

Simulations with Qibo



Weighted

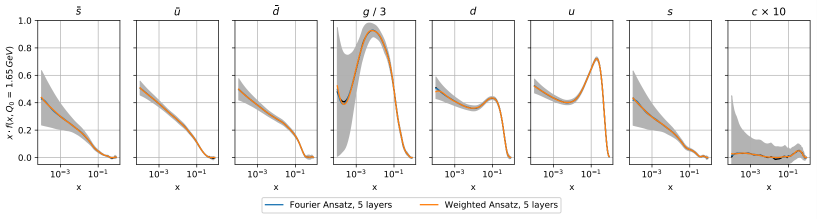
Single flavour fit		Multi flavour fit	
Layers (Params)	χ^2	χ^2	Layers (Params)
1 (32)	28.6328		1 (32)
2 (64)	1.0234	-	-
3 (96)	0.0388	0.1500	2 (72)
4 (128)	0.0212	0.0320	3 (112)
5 (160)	0.0158	0.0194	4 (152)
6 (192)	0.0155	0.0154	5 (192)

Fourier

Single flavour fit		Multi flavour fit	
Layers (Params)	χ^2	χ^2	Layers (Params)
1 (32)	900.694		1 (32)
2 (64)	57.2672	-	-
3 (96)	0.0410	47.4841	2 (72)
4 (128)	0.0232	0.0371	3 (112)
5 (160)	0.0165	0.0216	4 (152)
7 (192)	0.0156	0.0160	5 (192)

Simulations with Qibo

Example of multi-flavour qPDF fits using both Ansatze:



Stage 2: qPDFs in a global fit

N3PDF

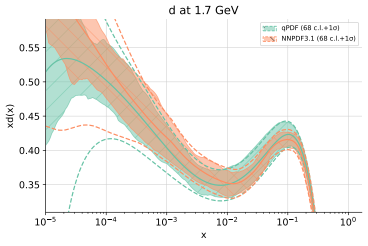
- `N3PDF` is a software used to extract PDFs from the experimental data at LHC
- `N3PDF` makes use of Neural Networks to model PDFs
- We have just replaced the NN with our quantum model
- The quantum PDF model needs less parameters in to provide comparable results
- The optimization procedure needs less iterations but a similar number of function evaluations

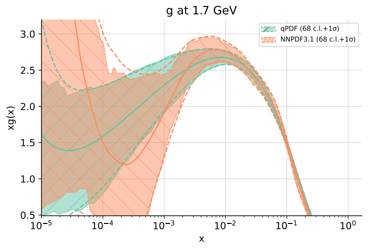
$$P = \int \mathrm{d}x_1 \, \mathrm{d}x_2 \, f_1^i(x_1, q^2) f_2^j(x_2, q^2) |M_{ij}(\{p_n\})|^2,$$

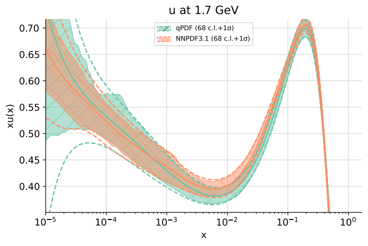
$$\chi^2 = \sum_{i,j}^{N_{\mathrm{dat}}} (D - P)_i \sigma_{ij}^{-1} (D - P)_j,$$

N3PDF results

PDF with experimental data

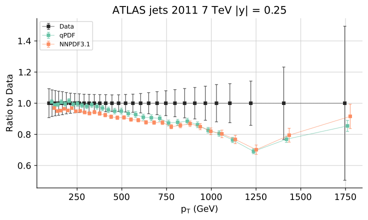


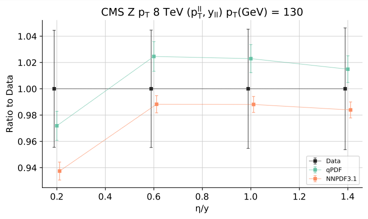


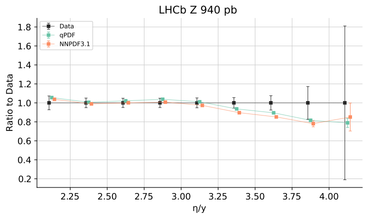


N3PDF results

Experimental data

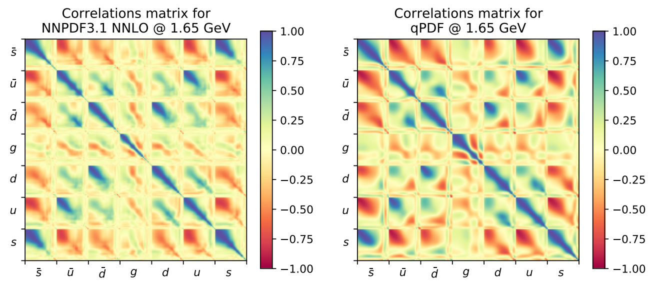






N3PDF results

Correlations



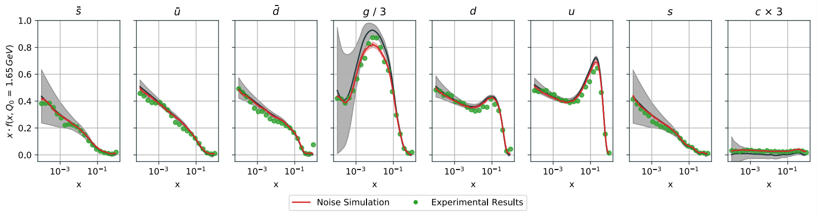
Some phenomenology

Channel	NNPDF3.1 NNLO	qPDF
ggH	31.04 ± 0.30 pb	31.77 ± 0.70 pb
$t\bar{t}H$	0.446 ± 0.003 pb	0.465 ± 0.015 pb
WH	0.133 ± 0.002 pb	0.135 ± 0.002 pb
ZH	0.0181 ± 0.0002 pb	0.0183 ± 0.0003 pb
VBF	2.55 ± 0.03 pb	2.55 ± 0.07 pb

Stage 3: qPDFs in a Q device

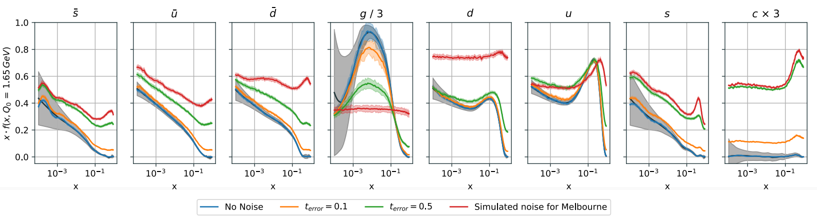
Experiments on a quantum computer

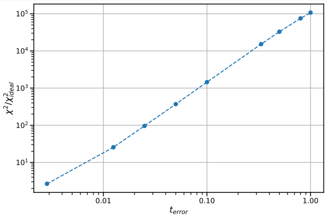
Real measurements from IBM Athens, 8k shots per x point:



Experiments on a quantum computer

Simulated measurements with noise for IBM Melbourne:





Conclusions

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- The quantum model with data re-uploading provides enough flexibility to fit PDFs
- This model provides fittings in agreement with experimental data
- The model can be successfully integrated in a general PDF calculator, `N3PDF`
- Quantum PDF model is successful when tested against phenomenological results
- Noise levels from current quantum computers are not enough to maintain the accuracy of this model

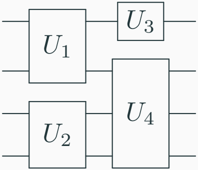
Thank you for your attention.

Rational for Variational Quantum Circuits

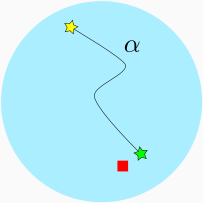
Rational:

Deliver variational quantum states $\rightarrow$ explore a large Hilbert space.

$$U(\vec{\alpha}) = U_n \ldots U_2 U_1$$



Near optimal solution



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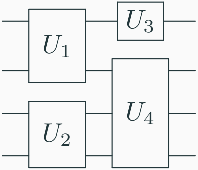

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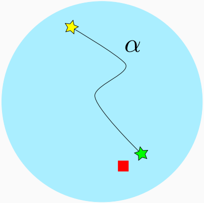


Idea:

Quantum Computer is a machine that generates variational states.

$\Rightarrow$ **Variational Quantum Computer!**

Solovay-Kitaev Theorem

Let $\{U_i\}$ be a dense set of unitaries.
Define a circuit approximation to V:

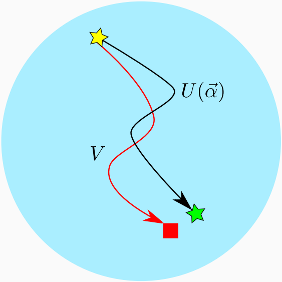
$$|U_k \dots U_2 U_1 - V| < \delta$$

Scaling to best approximation

$$k \sim \mathcal{O}\left(\log^c \frac{1}{\delta}\right)$$

where $c < 4$.

Optimal solution



$\Rightarrow$ The approximation is efficient and requires a finite number of gates.